Inflation and Stock Prices: A Rationality Story

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Abstract

This paper presents a rationality story of inflation and stock prices. Campbell and Vuolteenaho's (2004) VAR results advocate inflation illusions as the explanation for the positive association between inflation and the dividend yield. Contrary to their results, we find that a fully rational dynamic general equilibrium model, in which aggregate relative risk aversion depends upon inflation explicitly, can generate a positive correlation between the dividend yield and inflation of comparable size to its data counterpart. Our structural approach achieves an internal consistency of business cycle and financial variables in a general equilibrium framework. The VAR structure of our model solutions makes it possible to decompose the dividend yield into the long-run expected dividend growth rate and the discount rate components, so that their relative importance can be studied.

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The empirical relationship between inflation and stock prices has been a subject of enduring interest to academics, investment professionals, and the monetary policy makers. The leading practitioner model of equity valuation, the so-called "Fed model", implies that stock yields are highly positively correlated with inflation, a prediction borne out by the empirical evidence presented by Asness (2000,2003). The positive correlation between dividend yields and inflation contradicts the conventional wisdom that, equities, which represent claims to physical capital, ought to act as an inflation hedge. Three hypotheses have been put forward to explain this positive correlation. The first hypothesis is that, inflation, or the monetary authority's responses to inflation, damages the real economy, and in particular, the profitability of the corporate sector. In this case, the growth rate of real dividends declines in response to inflation, driving up the dividend-price ratio. The second hypothesis is that inflation makes investors more risk averse, driving up the equity premium, and thus the real discount rate. Brandt and Wang (2003) present a model of this sort. These two hypotheses can be modeled in a rational expectation environment, and we call them a rationality story.

Modigliani and Cohn (1979) propose a third hypothesis: namely inflation illusion. According to their hypothesis, stock market investors fail to understand the effect of inflation on nominal dividend growth rates and extrapolate historical nominal growth rates even in periods of changing inflation. From the perspective of a rational investor, this implies that stock prices are undervalued when inflation is high and overvalued when it is low.

Campbell and Vuolteenaho (2004, henceforth CV) use a VAR model to decompose the log dividend yield into three components, representing respectively the long-run expected dividend growth, the subjective risk-premium, and the mispricing component driven by inflation illusion. Their VAR results provide strong support to the importance of inflation illusion in explaining the positive association of dividend yield and inflation. However, due to the unobservability of long-run expected dividend growth and the risk premium, the mispricing component is a residual of both the forecast of future expected stock returns, and the subjective risk premium. As pointed out by Campbell and Ammer (1993), a shortcoming of such a VAR decomposition approach is that the results tend to overstate the importance of whichever component is treated as a residual.

In this paper, we take a distinctively different approach from CV(2004).

We build a New-Keynesian sticky-price model to study the relationship between inflation and dividend yields. Our model embeds the first hypothesis by examining how inflation, dividends and stock prices change in response to fundamental shocks under a given monetary policy rule. We also examine the importance of variations in the real discount rate, as stated in the second hypothesis, by adopting a preference specification where the relative risk aversion depends upon inflation explicitly. In our model, investors are averse to inflation risks. One way to motivate this inflation-induced variation in the relative risk aversion is through the anxiety consumers expressed about inflation in the recent survey of Shiller (1996). Our model can also be interpreted as a reduced form for an economy in which inflation proxies for the effects of an omitted real variable or some type of transaction cost which covaries with inflation.

In contrast to a VAR framework as in CV (2004), our structural approach achieves an internal consistency of business cycle and financial variables in a general equilibrium framework. It thus enables us to study the channels through which fundamental shocks affect both inflation and stock prices. Nevertheless, the VAR structure of our model solutions makes it possible to decompose the dividend yield into the expected dividend growth rate and the discount rate components, so that their relative importance can be studied.

The paper presents a rationality story of inflation and stock prices. Similar to CV (2004), the long-run expected dividend growth covaries with inflation in our model. As result, it is the positive correlation between the long-run discount rate and inflation which drives the positive association between the dividend yield and inflation. Specifically, high inflation makes investors more risk averse, driving up the real discount rate, and thus depressing the stock price. We find that a rationality story incorporated with the first two hypotheses is capable of generating a positive association of the dividend yield and inflation of comparable size to its data counterpart. The size of the positive correlation, and the relative importance of the long-run expected dividend growth versus discount rates depend upon the structural parameters of the model.

This paper belongs to an expanding literature on the influence of the macroeconomy on the stock market. Marshall (1992) studies inflation and asset returns in a monetary endowment economy with transaction costs. In our model, the production sector is nontrivial, and all the important variables, including consumption, dividends and inflation are endogenously determined. Brandt and Wang (2003) share our focus on the role of the real discount rate. However, both consumption and inflation are exogenous in their model. There has been considerable amount of work on the asset pricing implications of real business cycle models, such as Jermann (1998) and Boldrin, Christiano and Fisher (2001). However, the asset pricing implications of New-Keynesian sticky-price models, and in particular, the relationship between inflation and stock prices in such a framework, have not been well studied. This area is particularly interesting given the possible influence of monetary policy rules on these relations.

The paper is organized as follows: Section 1 describes a New-Keynesian sticky-price model, a standard setup except that investors are averse to inflation risks. Section 2 studies the theoretical implications of the model. Section 3 conducts the quantitative analysis and decomposes the dividend yield into the long-run dividend growth and discount rate components. Section 4 concludes.

1 The Model

In this section, we describe a standard New-Keynesian model. The model is standard except for the external habit formation specification similar to Campbell and Cochrane (1999).

1.1 Preferences

Representative households maximize expected lifetime utility of consumption, subject to a sequential budget constraint:

$$\max E_t \sum_{n=0}^{\infty} \beta^n \left[\frac{\left(C_{t+n} - X_{t+n}\right)^{1-\sigma} A_{t+n}^{\sigma} - 1}{1-\sigma} - \tau \frac{L_{t+n}^{\theta}}{\theta} + g\left(\frac{M_{t+n}}{P_{t+n}}\right) \right] \tag{1}$$

such that

$$C_t + \frac{M_t}{P_t} + f_{t+1}' V_t^f = W_t L_t + f_t' \left(V_t^f + D_t^f\right) + \frac{M_{t-1}}{P_t} + T_t. \tag{2}$$

The coefficient β is the subjective discount factor, C_t is real consumption at time t, and X_t is a subjective reference level against which consumption is measured. There are two restrictions on X_t. The reference level cannot exceed consumption, and is exogenous to the agent's choice set.

The coefficient σ measures the curvature of the representative agent's utility function with respect to its argument $C_t - X_t$. Relative risk aversion, which measures the curvature of the utility function with respect to consumption, is time-varying:

$$RRA_t = \sigma \frac{C_t}{C_t - X_t} = \sigma \Gamma_t,$$

where Γ_t is the reciprocal of the consumption surplus ratio as defined in Campbell and Cochrane (1999). We complete our description of the relative risk aversion by specifying a process for the deviation of $\log(\Gamma_t)$ from its steady state, denoted as γ_t. From now on, all lower case letters represent log-linearized deviations of corresponding variables from their steady state values. The process for γ_t is assumed as[2]:

$$\gamma_t = -\frac{b}{1-b}\left(c_t - c_{t-1}\right) + \phi \pi_t, \phi > 0, \tag{3}$$

where b measures the habit persistence based on consumption in the previous period, and a positive ϕ implies that the relative risk aversion increases with inflation. Given such a specification of γ_t, the deviation of the log of the marginal utility of consumption, ψ_t, can be expressed as:

$$\psi_t = -\frac{\sigma}{1-b} c_t + \frac{\sigma b}{1-b} c_{t-1} + \sigma \phi \pi_t + \sigma \varepsilon_{a,t}, \tag{4}$$

where $\varepsilon_{a,t}$, the logarithm of the preference shock A_t, is an i.i.d normal. When ϕ is set to 0, the marginal utility of consumption takes the same form as that derived from a utility specification of $U\left(C_t - bC_{t-1}\right)$.

Compared with a standard utility specification, inflation affects the marginal utility of consumption through two extra channels. First, due to the habit formation specification, the time-varying relative risk aversion depends negatively upon the growth rate of consumption, which varies with inflation. Second, the relative risk aversion depends positively upon inflation, thus driving up the marginal utility of consumption. The latter channel turns out to be important later on in accounting for the positive relation between the dividend yield and inflation.

[2]Specifying a process for γ_t is equivalent to Campbell and Cochrane's (1999) approach of specifying a process for the log consumption surplus ratio.

For the rest of the utility specification, the parameter τ reflects the degree of disutility from working, L_t is the labor supply at time t, θ indexes the degree of labor supply elasticity, and $g\left(\frac{M_t}{P_t}\right)$ represents the utility from holding the real money balance. In the budget constraint, f_{t+1}represents the vector of financial assets held at period $t+1$ and chosen at t; and V_t^f and D_t^f are vectors of asset prices and current period payouts, respectively; W_t represents the real wage, and T_t is the money injected in period t in a lump sum fashion.

The first-order conditions for consumption/saving, labor supply and money holdings are:

$$E_t\left[\beta\frac{\Psi_{t+1}}{\Psi_t}\frac{(1+R_t)}{P_{t+1}/P_t}\right] = 1, \tag{5}$$

$$\Psi_t = (C_t - X_t)^{-\sigma} A_t^{\sigma}, \tag{6}$$

$$W_t = \tau\frac{L_t^{\theta-1}}{\Psi_t}. \tag{7}$$

The money demand equation serves only to determine how much money the central bank needs to supply to clear markets given its interest rate target. This equation can be dropped when a monetary policy rule is present.

1.2 Production Technology and Price-Setting

We follow Bernanke, Gertler and Gilchrist (1998) in assuming a wholesale sector for production and a retail sector for pricing. Competitive firms produce wholesale goods, make decisions on how much output to produce and how much to invest. The retail sector differentiates these goods and sells it to an output aggregator, which combines heterogeneous products into Dixit-Stiglitz type final output, and then sells to households and wholesale production sector for consumption and investment.

1.2.1 Production Technology of Wholesale Sector

Wholesale goods are produced using the following technology:

$$Y_{w,t} = Z_t K_t^{1-\alpha} L_t^{\alpha}, \tag{8}$$

where the logarithm of the technology shock, Z_t, follows an AR1 process:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \text{where } \varepsilon_{z,t} \sim N\left(0, \sigma_z^2\right), 0 < \rho_z < 1.$$

The wholesale firms choose labor input optimally to maximize their profits. The maximization condition is:

$$\frac{P_{w,t}}{P_t} = \frac{W_t}{MPL_t}, \tag{9}$$

where the right hand side is the real wage over the marginal product of labor. Log-linearizing the above equation and substituting in equations (7) and (8) yields:

$$p_{w,t} - p_t = \frac{\theta - \alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta (1-\alpha)}{\alpha} k_t - \psi_t. \tag{10}$$

1.2.2 Phillips Curve

Retail firms purchase wholesale goods from the wholesale production sector, differentiate the products and sell them to the output aggregator. Since retail goods are heterogeneous, retail firms set prices taking the above demand curves as given.

We incorporate sticky prices into the model as Calvo (1983). We assume that at each period φ fraction of randomly chosen retail firms are free to set prices, while the rest have to set their prices according to $P_{j,t} = \frac{P_{t-1}}{P_{t-2}} P_{j,t-1}$. The derivations of the Phillips curve below is contained in the appendix.

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{\varphi\left[1-\beta(1-\varphi)\right]}{(1+\beta)(1-\varphi)} \left[\frac{\theta-\alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta(1-\alpha)}{\alpha} k_t - \psi_t\right]. \tag{11}$$

1.2.3 Investment Decisions

Wholesale firms make investment decisions given the above production technology. We assume convex adjustment costs in the investment technology[3]. Namely,

$$K_{t+1} = (1-\delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t,$$

where

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{\delta^\eta}{1-\eta}\left(\frac{I_t}{K_t}\right)^{1-\eta} + \frac{\eta\delta}{\eta-1}.$$

[3]The specification of covex adjustment cost is similar to Jermann (1998) and Boldrin, Christiano and Fisher (2001).

The parameter η measures the adjustment cost. When η approaches 0, adjustment cost is zero. The investment adjustment cost is infinite when η goes to infinity.

The maximization problem facing the wholesale firms is:

$$\max_{I_t} \sum_{n=0}^{\infty} E_t \left\{ \beta^n \frac{\Psi_{t+n}}{\Psi_t} \left[\frac{P_{w,t+n}}{P_{t+n}} Y_{w,t+n} - W_{t+n} L_{t+n} - I_{t+n} \right] \right\}.$$

Log-linearization of the first-order condition for investment yields:

$$\begin{aligned} q_t &= E_t\left(\psi_{t+1} - \psi_t\right) + \left[1 - \beta\left(1-\delta\right)\right]\left(p_{w,t+1} - p_{t+1} + y_{t+1} - k_{t+1}\right) + \beta E_t q_{t+1}, \\ q_t &= \eta\left(i_t - k_t\right). \end{aligned}$$

1.3 Monetary Policy Rule

We assume that the monetary authorities respond to the deviation of inflation from its mean and to the output gap. In particular, monetary policy is described by the following interest rate reaction function:

$$R_t = \rho_R R_{t-1} + \left(1 + \rho_\pi\right)\pi_t + \rho_y y_t + \varepsilon_{R,t}, \text{where } \varepsilon_{R,t} \sim N\left(0, \sigma_R^2\right), \qquad (12)$$

where $\rho_\pi, \rho_y > 0$. A positive ρ_π guarantees that the central bank adjusts the short-term nominal interest rate so that the targeted *ex post* real interest rate rises when inflation exceeds its target value, which is assumed to be the steady state rate of inflation. A positive ρ_y indicates a countercyclical monetary policy.

The goods market is in equilibrium when

$$Y_t = C_t + I_t.$$

2 Inflation and Stock Prices

In this section, we use the log-linearization methods to solve the model. The VAR structure of the model solutions makes it possible to decompose the log dividend yield into components related to future dividend growth rates and discount rates.

2.1 Model Solution and Asset Pricing Implications

We use the log-linearization method to solve the model. The model solution can be represented by a loglinear state space system, with the vector of state variables, s_t, following a first order autoregressive process with multivariate normal i.i.d impulses:

$$\mathbf{s}_t = \mathbf{\Lambda s}_{t-1} + \mathbf{B}\xi_t, \tag{13}$$

where the square matrix $\mathbf{\Lambda}$ governs the dynamics of the system. In the model economy considered here, $\mathbf{s}_t$ contains the nominal interest rate R_t, the inflation rate π_t, consumption c_t, the aggregate technology shock z_t, and the capital stock, k_{t+1}, which is determined based on information available at time t. The vector ξ_t contains normalized impulses, namely, $\left\{\frac{\varepsilon_{R,t}}{\sigma_R}, \frac{\varepsilon_{z,t}}{\sigma_\pi}, \frac{\varepsilon_{a,t}}{\sigma a}\right\}$.

This system provides us with the log of investment, i_t, the log of dividends, d_t, and the log of the marginal utility, ψ_t, as linear combinations of $\mathbf{s}_{t-1}$ and ξ_t. We derive expressions for the log of dividends and the dividend yield below to examine the relationship between inflation and stock prices.

The level of dividends, D_t, can be written as

$$D_t = C_t - W_t L_t \tag{14}$$

To derive an expression for the log of dividends, we first rewrite the Phillips curve as the log ratio of labor compensation out of output:

$$w_t + l_t - y_t = \frac{(1+\beta)(1-\varphi)}{\varphi[1-\beta(1-\varphi)]}\left(\pi_t - \frac{1}{1+\beta}\pi_{t-1} - \frac{\beta}{1+\beta}E_t\pi_{t+1}\right). \tag{15}$$

Using the above equation, we can easily write the log-linearized dividend as[4]:

$$\begin{aligned} d_t \;=\; & \frac{\widehat{c}\left(1-\widehat{c}+\widehat{d}\right)}{\widehat{d}}c_t - \frac{(1-\widehat{c})\left(\widehat{c}-\widehat{d}\right)}{\widehat{d}}i_t \\ & -\frac{\left(\widehat{c}-\widehat{d}\right)}{\widehat{d}}\frac{(1+\beta)(1-\varphi)}{\varphi[1-\beta(1-\varphi)]}\left(\pi_t - \frac{1}{1+\beta}\pi_{t-1} - \frac{\beta}{1+\beta}E_t\pi_{t+1}\right), \end{aligned}$$

where $\widehat{c}$ and $\widehat{d}$ are respectively the steady state ratio of consumption and dividend out of output. Substituting the model solutions for consumption,

[4]Details are contained in the appendix.

investment and inflation into this equation, the log of dividends can be rewritten as

$$d_t = \mathbf{d}_A \mathbf{s}_{t-1} + \mathbf{d}_B \xi_t.$$

Accordingly, based on equation (4), the log of the marginal utility can be written as

$$\psi_t = \psi_A \mathbf{s}_{t-1} + \psi_B \xi_t.$$

We proceed further to derive an expression for the dividend yields. We focus on the value of a conglomerate consisting of retail and wholesale firms. This makes sense since both sectors are owned by households. The value of the firm is

$$V_t = E_t \left[\beta \frac{\Psi_{t+1}}{\Psi_t} \left(D_{t+1} + V_{t+1} \right) \right],$$

and as a result, the log dividend yield is

$$d_t - v_t = -\left(E_t \psi_{t+1} - \psi_t \right) - \left(E_t d_{t+1} - d_t \right) + \beta E_t \left(d_{t+1} - v_{t+1} \right). \tag{16}$$

After substituting the solutions of d_t and ψ_t into the equation, we can solve for the log dividend yield as

$$\begin{aligned} d_t - v_t &= \widehat{\mathbf{y}}_A s_{t-1} + \widehat{\mathbf{y}}_B \xi_t, \text{ where} & (17) \\ \widehat{\mathbf{y}}_A &= \left(\psi_A + \mathbf{d}_A \right) \left(\mathbf{I} - \mathbf{A} \right) \left(\mathbf{I} - \beta \mathbf{A} \right)^{-1}, & (18) \\ \widehat{\mathbf{y}}_B &= \left(\psi_B + \mathbf{d}_B \right) - \left(\psi_A + \mathbf{d}_A - \beta \widehat{\mathbf{y}}_A \right) \mathbf{B}. & (19) \end{aligned}$$

The log dividend yield can be decomposed to examine the relationship between inflation and the two components, the future dividend growth and discount rate. The log-linear dynamic valuation framework of Campbell and Shiller (1988) show that

$$d_t - v_t = E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j} - E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j},$$

where $\triangle d$ denotes log dividend growth rate and r_t denotes log stock return. The decomposition says that changes in log dividend yields must be associated with changes in expectations of future dividend growth or real returns. The VAR structure of the model solutions makes it convenient to derive an expression for the second term in the above equation, namely,

$$E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j} = \mathbf{d}_A \left[(1-\beta) \left(\mathbf{I} - \beta \mathbf{A} \right)^{-1} \mathbf{A} - \mathbf{I} \right] \mathbf{s}_{t-1} + \left[(1-\beta) \mathbf{d}_A \left(\mathbf{I} - \beta \mathbf{A} \right)^{-1} \mathbf{B} - \mathbf{d}_B \right] \xi_t.$$

The coefficient in the regression of log dividend yield on inflation is:

$$\beta_\pi = \frac{cov\left(d_t - v_t, \pi_t\right)}{var\left(\pi_t\right)},$$

which can be further decomposed into the regression coefficients of the two components on inflation:

$$\beta_\pi = \frac{cov\left(E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}, \pi_t\right)}{var\left(\pi_t\right)} - \frac{cov\left(E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j}, \pi_t\right)}{var\left(\pi_t\right)}.$$

The coefficient β_π can also be decomposed according to the contribution of different exogenous shocks. Due to the moving average terms involved in computing the covariance between the dividend yield and inflation, the decomposition is most effectively done in numerical simulations.

3 Quantitative Predictions on Inflation and Stock Prices

The objective of the quantitative evaluation is to examine the model's ability to explain the positive association between the dividend yield and inflation, while maintaining reasonable business cycle implications such as output, consumption and investment volatility. The role of deep structural parameters, and in particular, monetary policy parameters, can also be studied in such a quantitative framework. We first start with the calibration of the model.

3.1 Calibration

The model parameters can be categorized into the following three groups:

3.1.1 Monetary Policy Rule Parameters

The monetary policy rule is characterized by the set of four parameters, $\{\rho_R, \rho_\pi, \rho_y, \sigma_r\}$. They are respectively set to $\{0.6, 0.5, 0.5, 0.002\}$. These parameter values are conventional in New-Keynesian models. Sensitivity analysis will be carried out to examine the significance of different monetary policy rules on the relation between inflation and stock prices.

3.1.2 Preference

The preference-related parameters consist of $\{\beta, \sigma, b, \phi, \sigma_a\}$. The subjective time discount rate, β, is set to 0.9986. Each model period is considered as one quarter. We set σ to 1 to guarantee balanced growth rates in the model[5]. The parameter b, which indexes the degree of habit persistence, is set to 0.83, similar to the value used in Jermann (1998). The parameter ϕ measures the degree of investors' aversion to inflation risk, and plays an important role in the relationship between inflation and stock prices. The value of ϕ is set to 3.5 in our benchmark calibration. We will vary the value of ϕ to examine the sensitivity of model implications to this important parameter. The parameter σ_a is set to 0.0001.

3.1.3 Production and Investment

There are seven production-related parameters: $\{\delta, \alpha, \rho, \sigma_z, \eta, \gamma, \theta, \tau\}$. The capital depreciation rate δ is 0.025. The constant labor share in a Cobb-Douglas production function is $\frac{2}{3}$. The persistence parameter of the technology shock is 0.96, and σ_z is set to 0.005. The parameter η stands for the inverse of the elasticity of the investment capital ratio with respect to Tobin's Q. We set η to $\frac{1}{0.23}$ to be comparable to the values used in Jermann (1998) and Boldrin, Christiano and Fisher (1999). When η goes to infinity, the investment adjustment cost approaches infinite. The parameter γ represents the degree of monopolistic competition in the economy. When γ approaches 1, the economy is close to perfect competition. We set γ to 0.3 in the benchmark case, and present some sensitivity analysis. The parameter θ, which describes the labor supply elasticity, is set to 2. The parameter τ and the steady state level of capital stock is set so that the consumption-output ratio is 0.75, and the fraction of labor used for production is 0.3.

3.2 Model Results

The benchmark model is able to generate reasonable business cycle and asset return statistics. Table 1 shows the model predictions on the relative volatility of consumption and investment relative to that of output, and the

[5]We assume the balanced growth rate to be 1. Since the correlations of log-linearized variables are the main focus of this paper, such an assumption is without loss of generality.

standard deviations and first order autocorrelations of nominal interest rate, inflation and the dividend yield.

Standard Deviations

	$\frac{\sigma(\triangle c)}{\sigma(\triangle y)}$	$\frac{\sigma(\triangle i)}{\sigma(\triangle y)}$	$std\,(R)$	$std\,(\pi)$	$std\,(dy)$
Benchmark	0.93	1.75	4.17	3.72	
Data	0.51	2.65	2.95	3.47	
	$corr_1\,(R)$	$corr_1\,(\pi)$	$corr_1\,(dy)$		
Benchmark	0.97	0.97			
Data	0.97	0.65			

Table 2 reports the regression of the dividend yield and its components on inflation in the same fashion as Table 1 in CV (2004). The model predicts a regression coefficient of the dividend yield on inflation of similar magnitude as what CV (2004) obtained using quarterly data.

Similar to CV(2004), the expected future dividend growth covaries with inflation. According to our model, it is the comovement of inflation and the expected discount rate component which explains the positive correlation between inflation and the dividend yield. The degree of the positive association between the two depends upon the structural parameters of the model.

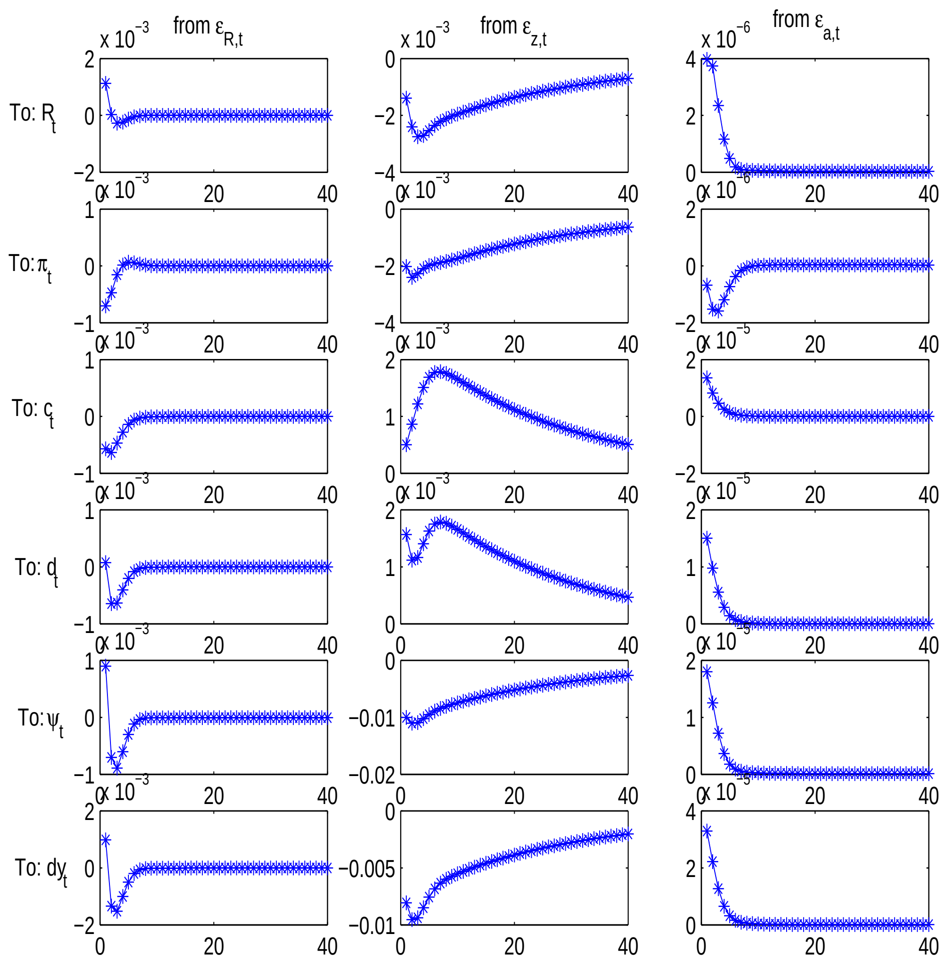
CV (2004) features a strong positive regression coefficient of the long-run expected dividend growth on inflation, thus other factors, especially the mispricing component, have to be strongly positively related with inflation to explain the positive association between the dividend yield and inflation.

	Benchmark		CV (2004)	
Dependent Variable	Coefficient on π_t	R^2(percent)	Coefficient on π_t	R^2(percent)
dy_t	3.39	97	4.01	7.19
$-E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j}$	-0.75	91	-11.25	94.78
$E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}$	4.14		15.26	

Figure 1 plots the impulse responses of both business cycle and financial variables in response to one standard deviation of the three fundamental shocks. In particular, we examine the impulse responses of nominal interest rate, inflation, consumption, dividends, the marginal utility of consumption, and the dividend-price ratio. In the figure, inflation declines in response to positive technology shocks, as a result of the declines in marginal costs.

Both consumption and dividends increase in response to positive technology shocks. However, the increase in the firm value, mainly initiated by the decline in the marginal utility of consumption, dominate the increase in dividends, resulting in declines in the dividend yield in response to technology shocks.

Figure 1: Impulse Responses



In our model, technology shocks are persistent, while the monetary policy and preference shocks are assumed to be temporary. As shown in the impulse responses, the impact of technology shocks on business cycle and financial variables is far more persistent than the other two types of shocks. It is no surprise that the former play a prominent role in the relationship between the dividend yield and inflation. Besides, the preference shocks lower inflation, raises the marginal utility of consumption, thus driving up the dividend yield, and resulting in a negative association between inflation and the dividend yield. In response to monetary policy shocks, inflation declines for 4 quarters, and the dividend yield increases on initial impact, but declines for the following 4 quarters. The impact of monetary policy shocks on these two variables is fairly small and short-lived as compared to that of technology shocks.

By sequentially assigning zero variance to one of the exogenous shocks, we are able to distinguish the importance of each type of exogenous shocks in the regression coefficient β_π. There is little positive association between the dividend yield and inflation when the variance of technology shocks are zero. This observation indicates that in our model, technology shocks are the primary forces behind the positive correlation between the dividend yield and inflation.

It is interesting to note that dividends increase more than consumption in the initial response to positive technology shocks. The changes in the labor compensation drive a wedge between the movement of consumption and dividend in a New-Keynesian sticky-price economy. Since the labor compensation typically covaries with inflation, the covariance between dividends and inflation caused by technology shocks is more negative compared tot he covariance between consumption and the latter.

3.2.1 Intuition: Why Dividend Yields Are Positively Related With Inflation?

As is evident now, the dividend yield and inflation are positively correlated because they both decline in response to positive technology shocks. This finding is along the same line as Fama's proxy hypothesis. Additional insight can be obtained if we consider several polar cases and conduct the sensitivity analysis.

First we consider the case when β approaches 1. In this case, equations (17) to (19) imply that $d_t - v_t \to d_t + \psi_t$, or equivalently the log-linearized firm value v_t approaches $-\psi_t$. The intuition is clear. The higher β is, the discount rate is relatively more important than dividends in determining the firm value.

This approximation relationship also enables us to decompose β_π as:

$$\beta_\pi \approx \frac{cov\left(d_t, \pi_t\right)}{var\left(\pi_t\right)} + \frac{cov\left(\psi_t, \pi_t\right)}{var\left(\pi_t\right)}.$$

As shown in our benchmark case, typically technology shocks drive the dividend up and inflation down, resulting in a negative covariance between the dividend and inflation. Note that the first term above is of the opposite sign with the regression coefficient of the long-run dividend growth component, $E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j}$, on inflation when β approaches 1. To the extent that high inflation coincides with negative technology shocks, the future dividend growth looks bright due to the low initial dividend level. This explains why the long-run expected dividend growth covaries with inflation in the above regression.

In order to obtain a positive relationship between the dividend yield and inflation, the covariance between the marginal utility of consumption and inflation must be a large positive value. In other words, the pricing kernel should be specified such that the covariance between ψ_t and π_t is high.

Further intuitions can be obtained if consider the Lucas tree economy, where d_t is equal to c_t. The dividend yield can be further approximated as follows[6]:

$$\begin{aligned} d_t - v_t &\approx c_t + \psi_t \\ &= (1-\sigma)\, c_t - \frac{\sigma b}{(1-b)} \left(c_t - c_{t-1}\right) + \sigma \phi \pi_t. \end{aligned}$$

Given the negative correlation between consumption and inflation, and the negative responses of inflation to positive technology shocks, a higher σ implies higher positive correlation between ψ_t and $\pi_{t.}$ However, the concern for balanced growth restricts our choice of σ to be 1.

When σ is set to 1, the covariance between the dividend yield and inflation is determined by the covariance between the consumption growth rate and inflation, the variance of inflation, and the parameter values of b and ϕ. If we set both b and ϕ to 0, as in all standard utility functions, the covariance between the dividend yield and inflation is close to zero.

As shown above, inflation declines in response to positive technology shocks. When ϕ is positive, the relative risk aversion, as defined in equation (3), decreases with inflation at the time of positive technology shocks.

[6] In this polar case, we set $\varepsilon_{a,t}$ to 0 to focus on the role of technology shocks.

The decline in the relative risk aversion boosts the firm value, leads to a decline in the dividend yield, and results in a higher positive association between the dividend yield and inflation.

By assuming investors' aversion to inflation risk and habit persistence, we introduce an additional channel through which inflation can affect the firm value and the dividend yield. One attractive feature of this specification is that since inflation decreases in response to positive technology shocks, the relative risk aversion is more countercyclical when ϕ is positive.

3.2.2 Sensitivity Analysis

The polar case where β approaches 1 illustrates the intuition behind our modelling strategy. When β declines, the dividend is more important in determining the firm value. As a result, the correlation between the dividend yield and inflation drops.

Table 3 reports the results of the sensitivity analysis from varying selected important structural parameters. When both b and ϕ are set to 0, there is a negative correlation between the dividend yield and inflation. When only b is set to zero, the regression coefficient of the dividend yield on inflation is only slightly smaller than the benchmark case. The results indicate that investors' aversion to inflation risks is the major reason for the positive relationship between dividend yield and inflation. A higher positive ϕ also leads to higher positive correlation between the two variables.

When we set the persistence parameter of technology shocks to zero, the correlation between inflation and the dividend yield decrease. Temporary technology shocks have a small impact on the firm value, whose negative correlation with inflation drives the positive association between inflation and the dividend yield.

Alternative Calibrations	Regression Coefficient of Dividend Yield's Component on Inflation		
	dy_t	$-E_t \sum_{j=0}^{\infty} \beta^j \triangle d_{t+1+j}$	$E_t \sum_{j=0}^{\infty} \beta^j r_{t+1+j}$
$b = 0, \phi = 0$	−0.12	−1.22	1.10
$b = 0$	3.28	−0.76	4.04
$\phi = 8$	7.62	−0.27	7.89
$\rho_z = 0$	1.07	−0.77	1.84
$\gamma = 0$	3.41	−0.73	4.14
$\rho_\pi = 5.5$	4.37	−11.14	15.51
$\rho_y = 10.5$	3.32	0.20	3.12

In the steady state of our model, the labor compensation as a fraction of output is equal to $\alpha\gamma$. Since dividend is equal to consumption minus labor compensation, the log of consumption is closest to that of dividend when γ approaches 0. Since labor compensation often covaries with inflation, the higher γ is, the stronger negative correlation between dividend and inflation, as compared with that between consumption and inflation. As a result, higher γ leads to lower positive association between the dividend yield and inflation.

With regard to monetary policy rules, a high ρ_π represents strong inflation-stabilizing stance of the monetary authorities. When ρ_π is set to 5.5, a higher value than the benchmark case, inflation declines only slightly. A stable inflation prevents the relative prices of some firms, in particular, those with sticky nominal prices, from rising too high. As a result, both consumption and dividends increase more in response to positive technology shocks as compared to the benchmark case. These strong responses can explain the large value of the regression coefficient of the long-run dividend growth component on inflation. The regression coefficient of the dividend yield on inflation is 4.37, higher than the benchmark case. When we set b to 0 while maintaining the high value of ρ_π, the size of the regression coefficient drops nearly by half. Although the variance of inflation is smaller under the stabilizing policy, the stronger negative covariance between inflation and the consumption growth rate leads to stronger positive association between the dividend yield and inflation. The regression coefficients in this case are strikingly similar to what CV (2004) obtained from their VAR framework.

A high ρ_y represents a strongly countercyclical monetary policy. When ρ_y is set to 10.5, the monetary authorities responds aggressively to positive technology shocks, which results in declines in both consumption and dividends. As a result, the regression coefficient of the long-run dividend growth rate on inflation is negative, while the coefficient of the dividend yield on inflation is similar to the benchmark case. The role of positive covariance between inflation and the long-run discount rate component is diminished in this case.

4 Conclusion

In this paper, we show that the positive association between the dividend yield and inflation, of comparable size to its data counterpart, can be rationalized in a standard New-Keynesian model where investors are averse to inflation. This specification is fairly general as any factor, which comoves with inflation, but moves in the opposite direction as technology shocks can take the place of inflation in the utility function.

CV (2004) advocate inflation illusions as the main reason behind the positive association between inflation and dividend yields. In our paper, the real discount rate increases relative to the real dividend growth rate because stocks are valued less when the investors are averse to inflation. CV (2004) acknowledges that some part of what they call mispricing may be in fact a second component of the risk premium, one that is common to all stocks, and thus does not appear in the cross-sectional measure of risk they use. Investors' aversion to inflation risk, as assumed in our model, seems to fit in this category. Our general equilibrium model presents an internally consistent description of the relationship between the dividend yield, inflation, and other business cycle and financial variables.

A Demand Facing Individual Retail Products

The output index Y_t is assembled using a constant returns to scale technology of the Dixit and Stiglitz (1977) form:

$$Y_t = \left\{ \int_0^1 Y_{j,t}^{\gamma} dj \right\}^{\frac{1}{\gamma}}, \tag{20}$$

where $0 < \gamma < 1$.

The output aggregator chooses the bundle of goods that minimizes the cost of producing a given quantity of the output index Y_t, taking as given the price $P_{j,t}$ of the good $Y_{j,t}$. The aggregator sells units of the output index at their unit cost P_t :

$$P_t = \left[\int_0^1 P_{j,t}^{\frac{\gamma}{\gamma-1}} dj \right]^{\frac{\gamma-1}{\gamma}}. \tag{21}$$

It is natural to interpret P_t as the aggregate price index. The aggregator's demand for each good $Y_{j,t}$ is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{\frac{1}{\gamma-1}} Y_t. \tag{22}$$

The pricing decision facing those price-setters is[7]:

$$\max_{P_t^*} \sum_{n=0}^{\infty} \left\{ (1-\varphi)^n E_t \left[\beta^n \frac{\Psi_{t+n}}{\Psi_t} \left(\frac{P_t^* \Pi_{m=0}^{n-1} \pi_{t+m} - P_{w,t+n}}{P_{t+n}} \right) Y_{j,t+n}^* \right] \right\},$$

where $Y_{j,t+n}^*$ is demand for good j given optimally chosen P_t^*.

Defining $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, log-linearizing equation (21) around the non-stochastic steady state and substituting in the optimal price P_t^* and equation (10), we obtain the Phillips curve.

[7]The derivations of the Phillips curve are identical to Christiano, Eichenbaum, and Evans (2001), except that in their model pricing decisions are made one period ahead of the realization of shocks.

B Derivation of the log of dividends

The log-linearized equation of (14) is:

$$\widehat{d}d_t = \widehat{c}c_t - \left(\widehat{c} - \widehat{d}\right)(w_t + l_t),$$

where as defined in the text, $\widehat{c}$ and $\widehat{d}$ are respectively the steady state ratio of consumption and dividend out of output, and $\widehat{c} - \widehat{d}$ is the steady state ratio of labor compensation out of output. Plug equation (15) and the log-linearized aggregate resource constraint:

$$y_t = \widehat{c}c_t + (1 - \widehat{c})\, i_t,$$

into the above equation, we can obtain the log-linearized dividend in the text.

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